The application of finite element modelling to guided ultrasonic waves in rails

R Sanderson and S Smith

There has been a rapid growth of interest in the use of guided ultrasonic waves for large-scale screening of components, predominately pipes, for corrosion[1,2]. These waves are analogous to Lamb waves in plates and exhibit dispersive behaviour. An important factor in enabling these waves to be used for non-destructive testing is knowledge of the dispersive properties, ie the variation of wave velocity with frequency for a given thickness of component. This is presented in graphical form as a dispersion curve.

An innovative numerical method for the calculation of dispersion curves of any prismatic section is proposed. To date, dispersion curves have been generated by solving analytical equations. However, as far as is known, there are no analytical solutions available for components with irregular cross-sections, such as rails. Dispersion curves for a rectangular steel bar and a steel rail are presented in this paper. Dispersion curves are essential information for the development of long-range ultrasonic inspection systems and the new technique opens up the possibility of developing a long-range ultrasonic inspection system for components with irregular cross-sections, such as rails.

The method described in this paper has been validated against available analytical solutions for simple geometries. The results from the numerical technique were found to agree closely with analytical solutions. An experiment has been carried out in a rectangular steel bar and predicted arrival times of the propagating waves have been found to agree with the times calculated using the group velocity dispersion curve obtained.

Ruth Sanderson joined TWI as a project leader in July 1998. Previously she studied for a BSc (Hons) in mathematics at Warwick University. Ruth is currently involved in various areas of finite element modelling including general stress analysis, weld modelling, fracture mechanics and long-range ultrasonics modelling.

Simon Smith joined TWI in 1989 with six years of finite element analysis experience applied to fracture mechanics. He studied for a BSc (Hons) and a PhD in Mechanical Engineering at Nottingham University. Simon has developed considerable experience with commercial FE software, and is now the TWI Technology Manager for Numerical Modelling. As a member of the Structural Integrity Technology Group, he has concentrated on developing methods for the prediction of the integrity of engineering components.

1. Introduction

1.1 Background and motivation

Dispersion curves are an invaluable part of the long-range ultrasonic inspection technique; they are an essential tool for understanding and describing the variation of the wave velocity of a particular mode with frequency. A long-range ultrasonic pulse can exhibit dispersive behaviour. From Fourier theory a pulse contains different components of frequency and, for a dispersive wave, each component will travel at a different velocity. As the pulse travels it will therefore appear to elongate, or disperse. The maximum overall amplitude of the pulse will also be diminished.

Long-range ultrasonic dispersion curves show either the phase or group velocity of an ultrasonic pulse for a given wave mode or modes and how dispersive each wave mode is at a particular frequency. The dispersion curves for components with regular cross-sections such as pipes, plates or rods are calculated by solving analytical equations. As far as is known, there are no available dispersion curve solutions for components of irregular cross-section, such as rails. The dispersion curve is used to choose a wave mode that is non-dispersive at the required frequency and also to choose a frequency at which the chosen, non-dispersive wave mode is isolated as much as possible from other similar wave mode excitations. This means that ultrasound received at a particular time can be associated with having a source at a particular position.

An innovative method developed at TWI uses a finite element analysis technique to calculate dispersion curves. The new method can be applied to prismatic structures of any cross-section, whereas the analytical method can only be applied to easily defined cross-sections, such as a pipe, which has axisymmetry. This work creates the potential for developing long-range ultrasonic inspection techniques for structures of any cross-section and any material. Long-range ultrasonic inspection systems are only commercially proven on pipes. This work is an essential element of the ongoing development of the Teletest®[3] system for pipe inspection and potentially, by using this new technique, a long-range ultrasonic rail inspection system could also be developed.

1.2 Wave modes and nomenclature

Guided ultrasonic waves can be excited in almost any structure. The more complicated the structure, though, the more complicated are the ways in which that structure will vibrate.

The main vibrations possible in a plate are called the anti-symmetric (a), symmetric (s) and shear horizontal (sh) wave modes. The symmetric and anti-symmetric wave modes in a plate are analogous to the longitudinal wave modes in a pipe and the shear horizontal wave modes are analogous to the torsional wave modes in a pipe. There are also flexural wave modes in a pipe.

Within these three basic types of vibration there are several ways in which a pipe can vibrate internally but still have the same overall displacement characteristic. For example, the arrangement of transducers in the Teletest® system is such that the longitudinal wave mode is preferentially excited. In the frequency range in which Teletest® operates, there are two instances of this wave, L(0,1) and L(0,2). The L stands for longitudinal. The first number in the brackets relates to the number of cycles of variation around the circumference of the pipe. As the longitudinal wave mode is axisymmetric it has no variation around the circumference, and this value is zero. The second number is the wave mode number. More complex displacements through the thickness have higher wave mode numbers[4].

1.3 Dispersion curve characteristics

The dispersion curve shows the phase or group velocity of an ultrasonic pulse plotted against the central frequency of the pulse. The slope of the dispersion curve determines how dispersive a particular wave mode can be. Dispersion results in the pulse becoming elongated, making interpretation of the data received difficult. Dispersion is generally an undesirable characteristic in current long-range ultrasonic inspection techniques.

1.4 Group velocity and phase velocity

The effect of velocity varying with frequency is that the different frequency components within a pulse travel at different velocities. The pulse changes shape and the amplitude diminishes as dispersion occurs. There are two ways of defining the velocity of an ultrasonic pulse. The phase velocity is defined as the velocity of points with constant phase angle. The group velocity is defined as the velocity of points with constant amplitude; it is the apparent speed with which the pulse travels through the material. Group velocity and phase velocity are equal for a non-dispersive wave so the pulse never changes shape. Reflections measured by the receiver probe can be used to calculate the position of the reflector using the information about the time of arrival of the reflected pulse and the group velocity of the wave.

Group velocity is relevant when a discrete group of waves, or pulse is travelling. From Fourier theory a group of waves can be expressed as a sum of sine and cosine functions with different frequencies. Therefore, within the group of waves, different frequencies are present and, if the gradient of the phase velocity dispersion curve is not zero, these different frequencies will have different phase velocities. Some components of the group of waves will move faster through the wave packet than others and hence dispersion will occur.

2. Finite element technique

Most commercial finite element software packages are able to predict natural frequencies. TWI uses ABAQUS[5] as a finite element solver. Natural frequency analyses can be used to find solutions for standing waves in a structure and their frequency of vibration. The mode shape of a standing wave in a prismatic section can be used to determine wavelength of that mode, see Figure 1. An algorithm has been written to analyse the displacement data and output the wavelength. The phase velocity of the wave can then be found via the simple relationship:

$$v = f \cdot \lambda \qquad [1]$$

Where: v = velocity
f = frequency of the wave mode
λ = wavelength of the wave mode

The velocity found from this is the phase velocity of the wave mode and can be plotted against the frequency. Each data point obtained from the finite element natural frequency analysis is a point on the phase velocity dispersion curve. A typical analysis will find hundreds of phase velocity dispersion curve points.

Standing waves have planes of zero axial displacement (or nodes in natural frequency modes). The model is therefore constrained at each end in the axial direction. No other constraints are needed. This constraint allows whole and half numbers of wavelengths to be found within the length of the model.

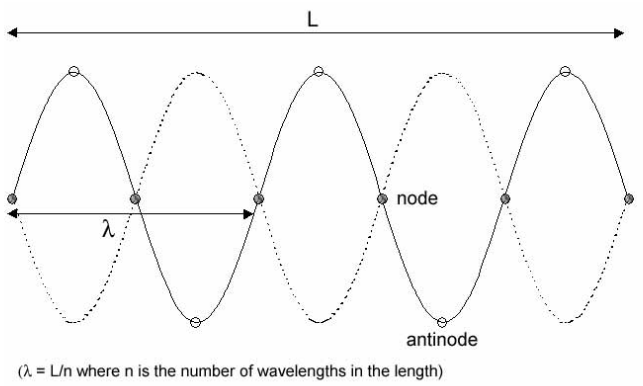

Figure 1. Standing wave diagram. The nodes stay in fixed positions, the string vibrates between the two extremes of the solid and dashed lines

For example, a model of length L=200 mm will find wavelengths of 2L, 2L/2, 2L/3, 2L/4, 2L/5, 2L/6, 2L/7, 2L/8, 2L/9, 2L/10, …*ie* 400 mm, 200 mm, 133.3 mm, 100 mm, 80 mm, 66.6 mm, 57.14 mm, 50 mm, 44.4 mm, 40 mm …This can potentially continue indefinitely but will be limited by the size of the elements in the model.

In some cases one model cannot capture the whole range of interest of the dispersion curves. Hence several models must be run. There is a trade-off between the length of model required in order to capture low frequencies and the number of elements required in order to capture high frequencies. In some cases efficiency can be improved by running several models of different lengths to obtain sufficient points over a wide range of frequency.

Each calculated wave mode is a point on the phase velocity dispersion curve via Equation [1]. In the case of a simple geometry such as a rod or pipe, there are only a few dispersion curve lines and, if many points are calculated, the dispersion curves can clearly be made out. However, in the case of more complicated geometries (such as rails) the number of possible wave modes for a given frequency increases substantially. It therefore becomes harder to clearly see the separate dispersion curves. TWI has developed algorithms to assist the identification of each curve when the points alone are not sufficient. The algorithms use displacement information from the model to separate out the dispersion curves from one particular wave mode, (such as a wave mode in a rail with only the head vibrating) from other wave modes (such as only the foot of the rail vibrating).

The finite element method for calculation of phase velocity dispersion curves allows interrogation of the model so that the displacement and stress distributions of each wave mode found can be examined.

3. Validation of the finite element technique

Initially, the finite element dispersion curve method was validated against analytical equations. The software package, Disperse[6] solves currently available analytical equations to obtain dispersion curves for rods, plates and pipes. A 3D model of a 40 mm-diameter steel rod was created and points calculated by the finite element method were found to agree closely with the lines calculated by Disperse, see Figure 2. A selection of displaced shapes obtained from the analyses is shown in Figures 3-5. The finite element dispersion curve method was also applied to a pipe using an axisymmetric model (in this case the model will find only the longitudinal modes). The results agree with experimental observations, in that if the predicted velocities are used, the reflectors are positioned correctly. The results were also validated against analytical equations and the points were found to agree, see Figure 6.

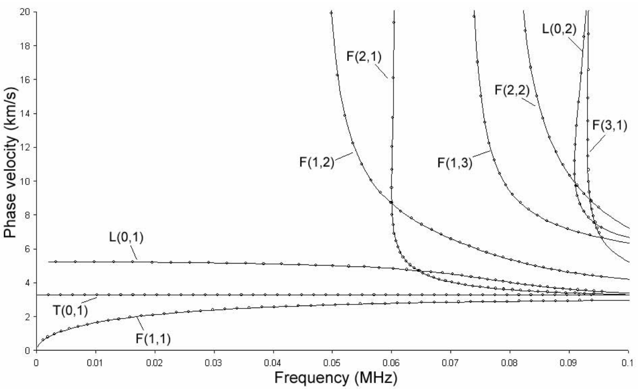

Figure 2. Dispersion curve comparison between Disperse (continuous lines) and FE results (points) for a 40 mm-diameter steel rod. L - longitudinal, F - flexural and T - torsional waves

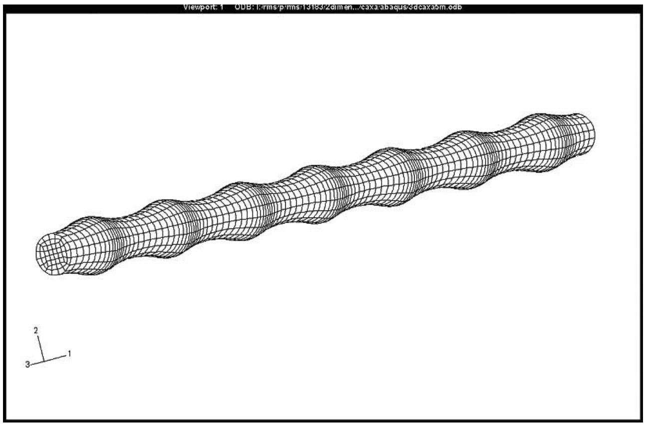

Figure 3. Example of longitudinal standing wave, L(0,1) in section of 40 mm-diameter steel rod

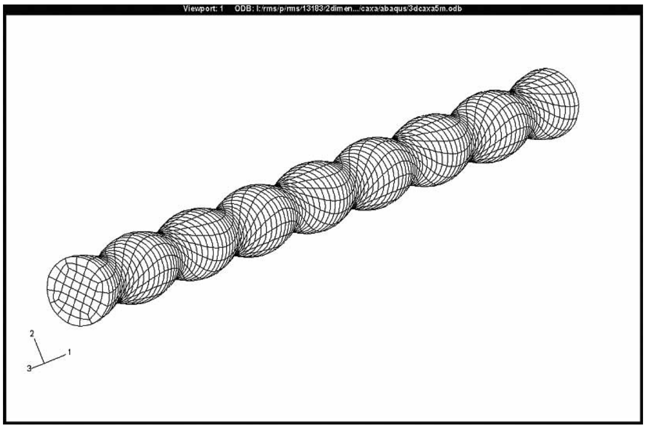

Figure 4. Example of torsional standing wave, T(0,1) in 40 mm-diameter steel rod

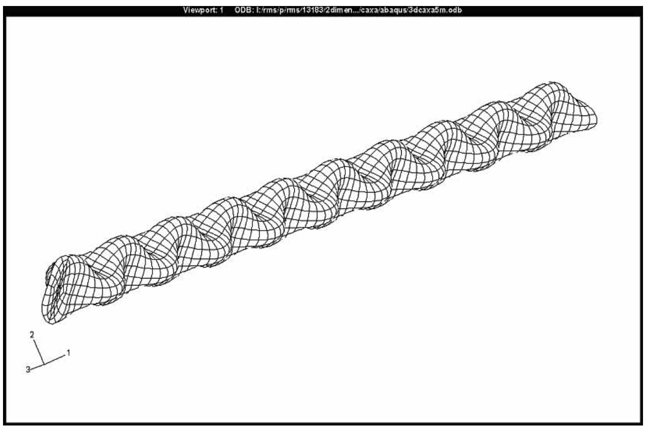

Figure 5. Example of flexural standing wave, F(2,1) in 40 mm-diameter steel rod

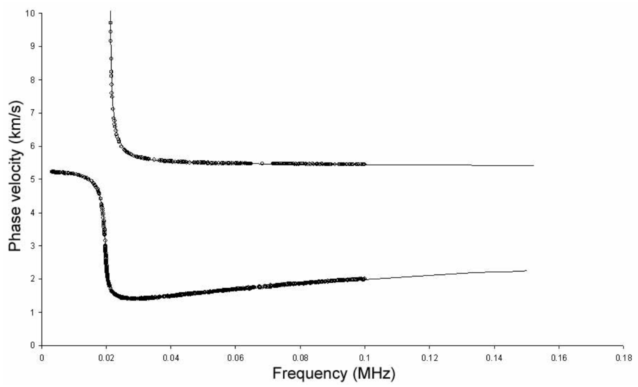

Figure 6. Comparison of dispersion curves of longitudinal modes in a 3" schedule 40 steel pipe calculated by FE method (points) and Disperse (continuous lines)

The numerical dispersion curve method was also applied to a 20 mm by 25 mm rectangular steel bar. The phase velocity dispersion curve for the bar obtained using the finite element analysis technique is shown in Figure 7. In the frequency range examined, there were four possible modes of vibration found. There were two flexural modes, with parallel dispersion curves. This is because the bar is not square in cross-section and therefore has different bending characteristics in the horizontal and vertical directions. If the vertical and horizontal flexural dispersion curves are normalised by multiplying the frequency by the thickness in the dominant direction of oscillation, they coincide. The other possible modes of vibration in the frequency range examined were the longitudinal and torsional wave modes.

The experimental set-up is shown in Figure 8. Two probes were attached to the bar, one to transmit an ultrasonic pulse, the other to receive the information. The input pulse was a 55 kHz five-cycle modulated wave. This was applied in a horizontal direction to the bar. This direction of excitation would result in preferential excitation of the horizontal flexural and torsional wave modes.

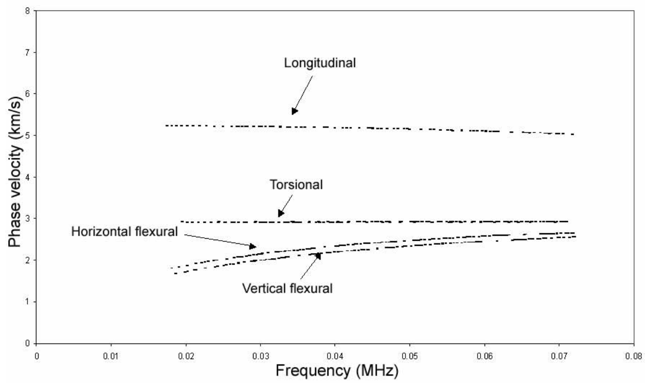

Figure 7. Finite element calculated dispersion curve for 20 mm by 25 mm steel bar

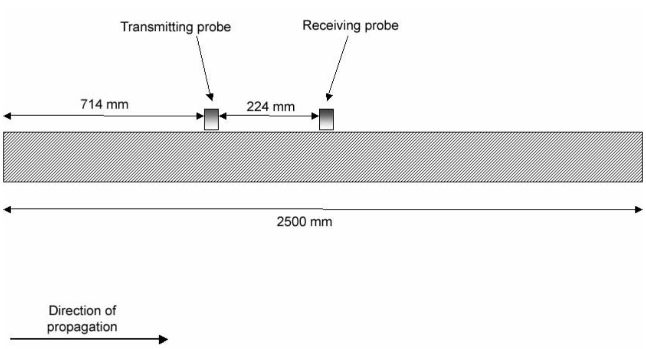

Figure 8. Experimental set up. A cross-section of the bar is shown (not to scale)

A plot showing the model predictions for arrival of the torsional and horizontal flexural wave modes and the trace obtained from the experiment are shown in Figure 9. The first pulse to arrive is the torsional and horizontal wave modes travelling together. The second pulse is the reflection of the two excited wave modes from the back end of the bar. The arrival times of all pulses and hence the predicted group velocities from the finite element group velocity dispersion curve predictions agree closely with the experimental results.

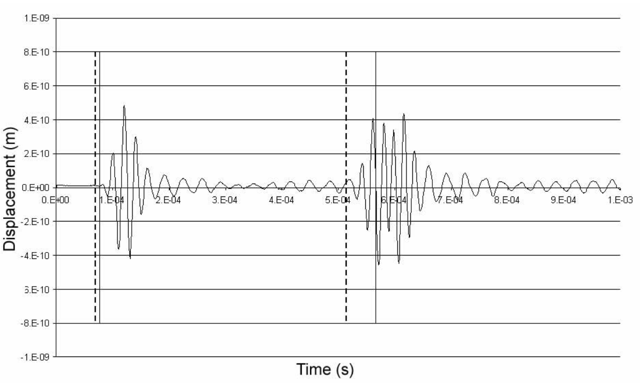

Figure 9. Comparison of experiment with finite element results (dashed vertical lines are predicted arrival times of horizontal flexural wave mode and continuous vertical lines are predicted arrival times of torsional wave mode)

4. Dispersion curve for a rail

There is a real need for long-range ultrasonic testing of rails and the analytical approach is not practically applied to the complicated rail geometry. Through providing a dispersion curve for a rail, the finite element technique opens up the possibility of developing a long-range ultrasonic inspection system for rails.

Using the finite element technique, a phase velocity dispersion curve for a steel rail was calculated. An example of the mesh used is shown in Figure 10. A selection of displaced shapes obtained from the analysis are shown in Figures 11-14. There were fewer points in the low frequency range so each of the displaced shapes could be examined in order to determine which mode they belonged to. It is also helpful to know that it is impossible to find two separate displaced shapes with the same number of wavelengths in the model, belonging to the same mode. The phase velocity dispersion curves calculated are shown in Figure 15. The more easily tracked, low frequency dispersion curves are indicated by continuous lines. In the frequency range of 20-50 kHz the number of wave modes increases dramatically. This makes it harder to track the dispersion curve lines, however lines can just be seen to be sweeping in steeply and levelling off.

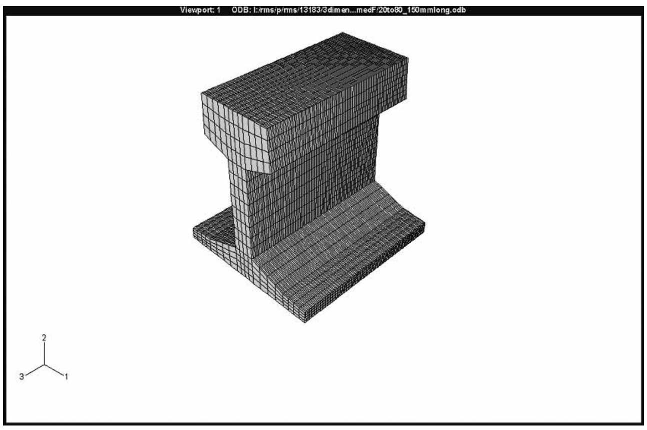

Figure 10. Example mesh of rail used to calculate dispersion curve

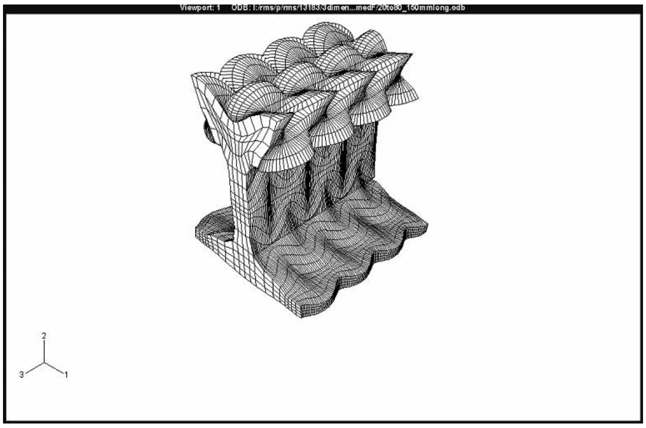

Figure 11. Example wave mode in a rail

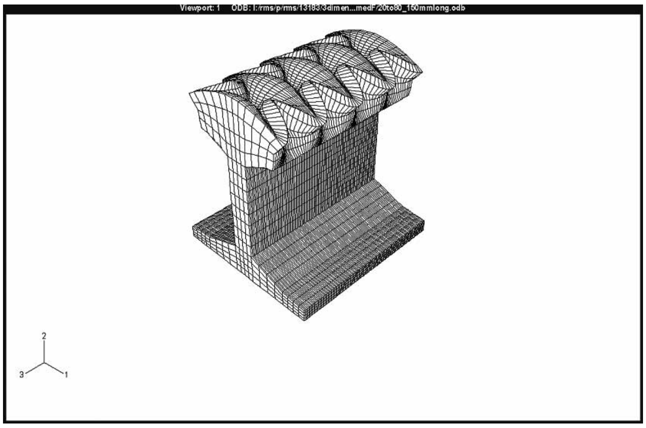

Figure 12. Example wave mode in a rail. The excitation is predominately in the head of the rail

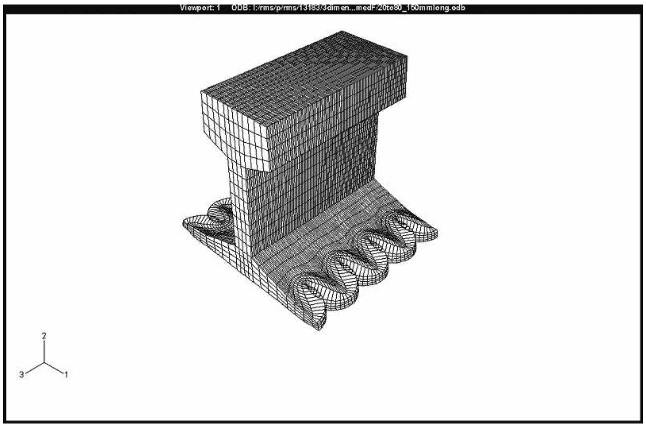

Figure 13. Example wave mode in a rail. The excitation is predominately in the foot of the rail

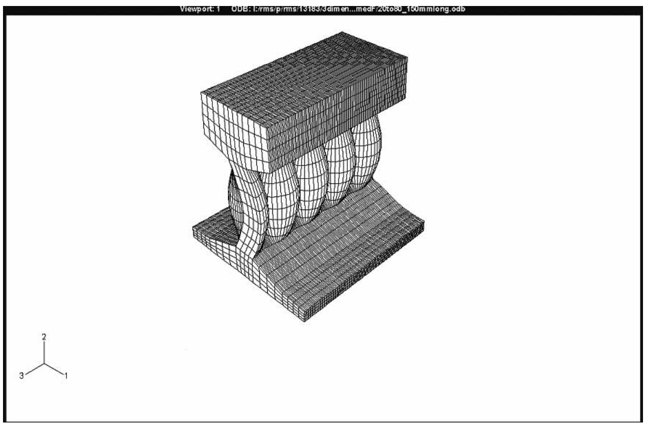

Figure 14. Example wave mode in a rail. The excitation is predominately in the mid-section of the rail

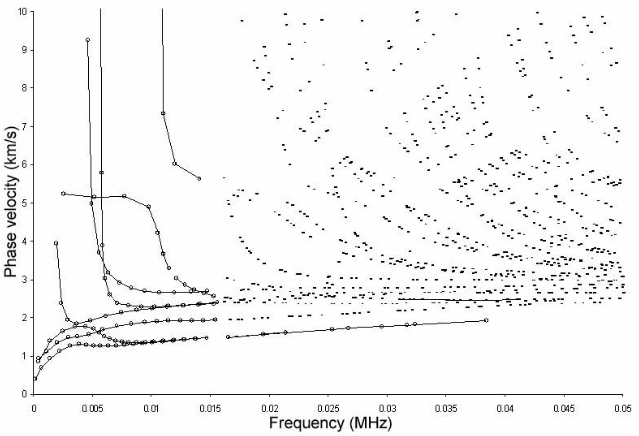

Figure 15. Dispersion curves for a rail

5. Discussion

Since some modes have been found to only travel in the head of the rail, they could be more easily isolated. The excitation would be applied purely to the head of the rail. These wave modes could also be particularly useful for isolating flaw detection to the head of the rail. However, interaction of ultrasonic pulses with flaws can result in mode conversion. Mode conversion would be undesirable if a large number of modes are excited, as a result of hitting the flaw, so that the resulting signal can not be interpreted. However, certain mode conversions could be useful in detecting the flaw.

In the long-range ultrasonic inspection of pipes, mode conversion is often relied upon to determine that a flaw exists, rather than a legitimate feature of the component. The foot of the rail is clamped to the sleepers; this constraint may therefore suppress mode conversion from modes travelling purely in the head of the rail to other modes which have an element of vibration in the foot of the rail. It is possible that some entire body modes of vibration in rails could suffer from less mode conversion than other modes; these modes could prove to be useful for finding flaws anywhere in the rail. TWI is currently carrying out finite element wave propagation analyses to study the reflection and transmission of modes from flaws. These will help to identify the modes suitable for clear detection of flaws.

The ability to detect a flaw is improved by having shorter wavelengths in relation to the flaw size. Since wavelength decreases with increasing frequency, this would indicate that better flaw detection would be achieved using higher frequencies. However, the points on the dispersion curve for the steel rail demonstrate that at frequencies greater than about 20 kHz ($\lambda \sim 50$ cm) there are a large number of possible modes of vibration. Generally speaking, the greater the number of modes present, the more difficult it becomes to interpret received signals. Consequently, although ideally, as high a frequency as possible should be used to detect the smallest possible flaws, in practice a frequency must be used such that flaws of the target size may be detected reliably.

Factors governing the minimum size of flaw which can be detected using guided waves are complex and beyond the scope of this paper. To address this issue, interaction of modes with flaws will be the subject of further modeling work, so at this stage it is not possible to give guidance about the minimum size of flaw which might be detectable.

6. Conclusions

The results from the dispersion curve calculation technique have been validated against analytical solutions for a steel rod and a steel pipe. The dispersion curve calculation technique has been validated with an experiment by comparing predicted arrival times from the group velocity dispersion curve with actual arrival times recorded by a receiving probe in the experiment.

- A method has been developed using finite element analysis to calculate dispersion curves of any prismatic cross section
- The finite element method for calculating dispersion curves has been validated against analytical solutions and confirmed by experimental observations.
- Experimentally measured group velocities in a rectangular steel bar have been accurately predicted using a dispersion curve generated by the new method.
- A significant amount of information has been generated relating to the dispersion curves for a steel rail. Wave modes have been identified which are isolated to the head, foot or mid-section of the rail. These modes may potentially be used to detect flaws in the relevant parts of the rail.

References

1. Mudge P J, Lank A M, Alleyne D N: ‘A long range method of detection of corrosion under insulation in process pipe work’ 5th European Union Hydrocarbons Symposium, Edinburgh, 26-28 November 1996.
2. Mudge P J and Lank A M: ‘Detection of corrosion in pipes and pipelines’, ASNT International Chemical and Petroleum Industry Inspection Technology Topical Conference V, Houston, Texas, 16-19 June 1997.
3. Mudge P J: ‘Field application of the Teletest® long-range ultrasonic testing technique’. *Insight* 2001 43 (2) 74-77.
4. Silk M G and Bainton K F: ‘The propagation in metal tubing of ultrasonic wave modes equivalent to lamb waves’, Ultrasonics, 17, 11-19 (1979).
5. ABAQUS Version 6.2, HKS Inc, Pawtucket, RI, USA.
6. Pavlakovic B and Lowe M: Disperse v2.0 User’s Manual, June 2000.